Chi Squared Tests (Contingency Tables) (From OCR 4734)

## Q1, (Jan 2008, Q6)

(i)
(ii)
$\mathrm{H}_{0}$ : Gender and shade are
$\left(\mathrm{H}_{1}:--\right.$ are not independent
$3.02^{\left(14.02^{-1}+14.98^{-1}\right)+}$
$6.12^{2}\left(17.88^{-1}+19.12^{-1}\right)$
$+3.1^{2}\left(26.1^{-1}+27.9^{-1}\right)$
$=6.03$
EITHER: CV 5.991
$6.03>5.991$, reject $\mathrm{H}_{0}$ and accept that gender and shade are not independent
OR: $\mathrm{P}\left(\chi^{2}>6.03\right)=0.049$
$<0.05$, reject $\mathrm{H}_{0}$ and accept that gender and shade are not independent
(iii)

|  | $\mathrm{G}_{1}$ | $\mathrm{G}_{2}$ | $\mathrm{G}_{3}$ |
| :--- | :---: | :---: | :---: |
| O | 29 | 37 | 54 |
| E | 40 | 40 | 40 |
| $121 / 40$ | $+9 / 40+196 / 40$ |  |  |
| $=8.15$ |  |  |  |
| Using df $=2$ |  |  |  |
| $2.5 \%$ tables, $1.7 \%$ |  |  |  |
| calculator |  |  |  |

Q2, (Jan 2010, Q7)
(i) $\mathrm{H}_{0}$ :Vegetable preference is independent of gender $\mathrm{H}_{1}$ : All alternatives
$\begin{array}{rlll}\text { E-Values } & 26 & 16.25 & 22.75 \\ & 22 & 13.75 & 19.25 \\ \chi^{2} & =5^{2}\left(26^{-1}+22^{-1}\right)+ & 7.25^{2}\left(16.25^{-1}+13.75^{-1}\right) \\ & =9.641 & & +2.25^{2}\left(22.75^{-1}+19.25^{-1}\right)\end{array}$
$9.64>5.991$
Reject $\mathrm{H}_{0}$, (there is sufficient evidence at the $5 \%$ that) vegetable preference and gender are not independent
(ii)
$\left(\mathrm{H}_{0}\right.$ : Vegetables have equal preference
$\mathrm{H}_{1}$ : All alternatives)
Combining rows: $48 \quad 30 \quad 42$
E-Values: $\quad 40 \quad 40 \quad 40$
$\chi^{2}=\left(8^{2}+10^{2}+2^{2}\right) / 40$
$=4.2$
$4.2<4.605$
Do not reject $\mathrm{H}_{0}$, there is insufficient evidence at the $10 \%$ significance level of a difference in the proportion of preferred vegetables

For both hypotheses
At least one correct
All correct
Correct form of any one
All correct
ART 9.64
OR: $\mathrm{P}(\geq 9.641)=0.00806<0.05$

8

Or equivalent

At least two correct
All correct

Ft $X^{2}$. Can be assertive.

Ft $X^{2}$

For combining

M1
A1
M1
A1
M1
A1

| $\begin{aligned} & \text { M1 } \\ & \text { A1 } 2 \end{aligned}$ | Or equivalent |
| :---: | :---: |
| B1 |  |
| M1 | At least two correct |
| A1 | All correct |
| A1 |  |
| B1 |  |
| M1 |  |
| $\begin{aligned} & \text { A1 } \\ & \text { B1 } \end{aligned}$ | Ft $X^{2}$. Can be assertive. |
| M1 |  |
| A1 $\sqrt{ }$ | Ft $X^{2}$ |
| M1 | For combining |
| A1 |  |
| M1 |  |
| A1 |  |
| M1 |  |
| A1 6 (15) |  |


| B1 |  | For both hypotheses |
| :---: | :---: | :---: |
| M1 |  | At least one correct |
| A1 |  | All correct |
| M1 |  | Correct form of any one |
| A1 |  | All correct |
| A1 |  | ART 9.64 |
| M1 |  | OR: $\mathrm{P}(\geq 9.641)=0.00806<0.05$ |
| 8 |  |  |
| M1 |  |  |
| A1 |  |  |
| M1 |  |  |
| A1 |  |  |
| M1 |  | $\mathrm{OR}: \mathrm{P}(\geq 4.2)=0.122>0.10$ |
| A1 | 6 |  |
| [14] |  | AEF in context |

(i) $\mathrm{H}_{0}$ : There is no association between the area in which a shopper lives and the day they shop
( $\mathrm{H}_{1}$ : All alternatives)
$\begin{array}{lll}\text { E-Values } & 27.3 & 14.7\end{array}$
$37.7 \quad 20.3$
$X^{2}=(4.3-0.5)^{2}\left(27.3^{-1}+37.7^{-1}+14.7^{-1}+20.3^{-1}\right)$ $=2.606$
Compare with 2.706 Do not reject $\mathrm{H}_{0}$. There is insufficient evidence of an association.
(ii)

SR: If $\mathrm{H}_{0}$ association, lose $1^{\text {st }} \mathrm{B} 1$ and last M1A1

Conclusion the same since critical value > 2.706
(and test statistic unchanged)
B1
M1
A1
M1 ft
A1
A1
M1
A1

Q4, (Jan 2011, Q7)
(i) In a $2 \times 2$ contingency table
(ii) $\mathrm{H}_{0}$ : Vaccine type and outcome are independent
$\mathrm{H}_{1}$ : They are not independent
E-values: $10.81 \quad 12.19$
318.19358 .81
$\chi^{2}=7.69^{2}\left(10.81^{-1}+12.19^{-1}+318.19^{-1}+358.81^{-1}\right)$

$$
=10.67
$$

$\mathrm{CV}=6.635$
$10.67>$ CV
Reject $\mathrm{H}_{0}$, there is sufficient evidence at the $1 \%$ significance level that the outcome of the test depends on the vaccine used

The results is significant at a level less than $1 / 2 \%$, so the evidence is very strong

## B1 $\mid$ SR difference in proportions

 B1 define and evaluate $p_{1}$ and $p_{2}$ with $\mathrm{H}_{0}$B1 for $p=0.42$
M1A1 for $z= \pm 1.827$ or 1.835 (no pe)
M1A0 Max 5/8
At least one E value correct (M1)
All correct(A1)
At least one $X^{2}$, no or wrong cc, (M1FtE)
All correct (A1); 2.606 or 2.61 (A1)
Or use calculator ( $p=0.106$ ) SR: B1
if no explicit comparison, as Q2
SR: If $\mathrm{H}_{0}$ association, lose $1^{\text {st }} \mathrm{B} 1$ and last M1A1

OR from $z= \pm 2.17, \mathrm{SR}$

| B1 1 | Or equivalent Accept df=1 |
| :---: | :---: |
| B1M*dep | Accept omission of $\mathrm{H}_{1}$ |
| M1 | 1 correct E value |
| A1 | Accept 1 dp |
| M1 | 1 correct $\chi^{2}$ value ft E values |
| M1 | Using Yates' correctly |
| A1 | Accept 10.7 |
| B1 |  |
| M1 |  |
| $\begin{aligned} & \mathrm{A} 1 \sqrt{ } \\ & \operatorname{dep}^{*} \mathrm{M} \end{aligned}$ | $\sqrt{ } 10.67$ |
| $\begin{gathered} \mathrm{A} 1 \sqrt{ } 10 \\ {[11]} \end{gathered}$ | Sensible comment. $\sqrt{ } 10.67$ |

## Q5, (Jun 2012, Q2)

(i) $\quad \mid \mathrm{H}_{0}$ : no association between sex and artist preferred $H_{1}$ : some association between sex and artist preferred

| EXPECTED | Monet | Renoir | Degas | Cézanne |
| :--- | :---: | :---: | :---: | :---: |
| Male | 12.13 | 28 | 13.07 | 16.8 |
| Female | 13.87 | 32 | 14.93 | 19.2 |


| CONTRIB’N | Monet | Renoir | Degas | Cézanne |
| :--- | :---: | :---: | :---: | :---: |
| Male | 1.4081 | 0.3214 | 1.8626 | 0.2881 |
| Female | 1.2321 | 0.2813 | 1.6298 | 0.2521 |

$X^{2}=7.28$
Refer to $\chi_{3}{ }^{2}$
Critical value at $10 \%$ level $=6.251$
Result is significant
There is evidence to suggest that there is some association between sex and artist preferred

NB if $\mathrm{H}_{0} \mathrm{H}_{1}$ reversed, or 'correlation' mentioned, do not award first B1 or final E1

B1 For both hypotheses in context

For expected values
(to 2 dp where appropriate)
(allow A1 for at least one row or column correct)

For valid attempt at $(\mathrm{O}-\mathrm{E})^{2} / \mathrm{E}$
For all correct (to 2 dp ) and presented in a table or clear list. (Allow A1 for at least one row or column correct)

Allow 7.27
for 3 deg of f
CAO for cv No FT from here if wrong or omitted, unless $p$-value used instead FT their $X^{2}$

For correct (FT their $X^{2}$ ), non-assertive conclusion, in context.

NB:These three marks cannot be implied by a correct final value of $X^{2}$
www

B1 for $p$-value $=$ 0.0636

| Monet: More females and fewer males than expected |
| :--- |
| prefer Monet, as indicated by large contribution(s) (of |
| 1.4081 and 1.2321 ). |
|  |
| Renoir: Preferences are much as expected, as indicated by |
| small contributions. |
| Degas: Fewer females and more males than expected |
| prefer Degas, as indicated by large contribution(s) (of |
| 1.8626 and 1.6298 ). |
| Cézanne: Preferences are much as expected, as indicated |
| by small contributions. |


| E1* <br> E1dep* | FT their table of contributions | NB MAX 3/6 for <br> answers not referring <br> to contributions <br> (explicitly or <br> implicitly). |
| :---: | :--- | :--- |
| E1 |  | E1* <br> depE1* |
| E1 |  | SC1 Renoir and <br> Cézanne have correct <br> comments for both <br> but without referring <br> to contributions |
| $[6]$ |  |  |

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| Q6, (Jun 2013, Q6) |  |  |  |
| :---: | :---: | :---: | :---: |
| (i) | $\begin{array}{\|rr\|r\|} 17.5 & 4.2 & 6.3 \\ 32.5 & 7.8 & 11.7 \end{array}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { A1 } \\ & {[3]} \end{aligned}$ | $\mid \text { eg } 50 \times 28 \div 80$ <br> At least 2 correct. All correct. |
| (ii) | The E value of $4.2<5$ Combine Biology and Chemistry (both sciences). | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \\ & \text { [2 } \end{aligned}$ | Need not mention 4.2 <br> May need to look at (iii) to see which subjects combined. |
| (iii) | Ho: Subject and sex are independent <br> $\mathrm{H}_{1}$ : They are not independent <br> 21.76 .3 <br> 40.311 .7 $\begin{aligned} \chi^{2} & =(4.7-0.5)^{2}\left(21.7^{-1}+6.3^{-1}+40.3^{-1}+11.7^{-1}\right) \\ & =5.558 \ldots \\ (v & =1) \end{aligned}$ <br> (a) $2 \frac{1}{2} \% \mathrm{CV}=5.024$ <br> $5.558>\mathrm{CV}$ or in CR and reject $\mathrm{H}_{0}$ | B1 <br> B1 <br> M1M1 <br> A1 <br> B1 <br> M1 | oe. NOT 'variables', 'they' etc <br> or 17.510 .5 <br> 32.519 .5 if C/A combined. <br> No Yates(inc $v>1$ ) or incorrect Yates (eg no modulus) M1M0. allow 6.96 or 6.79 <br> Chem./Art combined B1B1M1M1A0B1M1A0. (TS = 3.75) <br> ft TS \& CV. Correct first conclusion. If C/A prob. accept $\mathrm{H}_{0}$. |
|  | $\begin{gathered} (\beta) \mathrm{P}\left(\chi^{2}{ }_{1} \geq 5.558\right)=0.0184 \\ <0.025 \text { and reject } \mathrm{H}_{0} \end{gathered}$ | $\begin{aligned} & \hline \text { B1 } \\ & \text { M1 } \\ & \hline \end{aligned}$ |  |
|  | There is significant evidence that subject and sex are not independent | A1 $[8]$ | cwo. NOT over-assertive. Thus no or incorrect Yates can score $\max 6 / 8$ <br> B1B1M1M0A1B1M1A0. |

## Q7, (Jun 2016, Q2)

$\mathrm{H}_{0}$ :there is no assoc. between party and opinion, $\mathrm{H}_{1}$ :there is assoc. between $\mathrm{p} / \mathrm{o}$. Expected frequencies
$45,18,27,20,8,12,35,14,21$
$\frac{(58-45)^{2}}{45}+\ldots .+\frac{(33-21)^{2}}{21}$
30.48

| B1 | For both.Allow indpt. etc. |  |
| :--- | :--- | :--- |
| M1 |  |  |
| A1 |  |  |
| M1 | At least one correct term; at least 7 terms. | If classes combined, all 6. |
| A1 | Allow awrt 30.5 |  |
| M1 |  |  |
| A1 | CWO |  |
| [7] |  |  |

Q8, (Jun 2017, Q3)
$\mathrm{H}_{0}$ : there is no assoc between hair/eyes colours. $\mathrm{H}_{1}$ : there is assoc ........
Es 30.16, 27.84, 21.84, 20.16
$(|36-30.16|-0.5)^{2 / 30.16}+\ldots .$.
allow this mark if no Yates' correction.(5.61) $\quad 0.945+1.306+1.024+1.414$
4.69
$\mathrm{CV}=3.841$
$4.69>3.841$, reject $\mathrm{H}_{0}$,
There is evidence of an assoc. between
hair/eye colours.

|  |  |
| :--- | :--- |
| allow this mark if no Yates' correction.(5.61) | $0.945+1.306+1.024+1.414$ |
| ft TS and CV <br> cwo. Contextualised. |  |

